

Leonard Gamberg*

Department of Physics and Astronomy, University of Pennsylvania, David Rittenhouse Labs, 209 South 33rd Street, Philadelphia, PA 19104

and

Department of Physics and Astronomy, Tufts University, Medford, MA 02155

Gary R. Goldstein†

Department of Physics and Astronomy, Tufts University, Medford, MA 02155

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The axial vector and tensor charge, defined as the first moments of the forward nucleon matrix elements of corresponding quark currents, are essential for characterizing the spin structure of the nucleon. However, the transversity distribution and thus the tensor charge decouple at leading twist in deep inelastic scattering, making them hard to measure. Additionally, the non-conservation of the tensor charge makes it difficult to predict. There are no definitive theoretical predictions for the tensor charge, aside from several model dependent calculations. We present a new approach that exploits the approximate mass degeneracy of the light axial vector mesons ($a_1(1260)$, $b_1(1235)$ and $h_1(1170)$) and uses pole dominance to calculate the tensor charge. The result is simple in form. It depends on the decay constants of the axial vector mesons and their couplings to the nucleons, along with the average transverse momentum of the quarks in the nucleon.

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The spin composition of the nucleon has been intensely studied and has produced important and surprising insights, beginning with the revelation that the majority of its spin is carried by quark and gluonic orbital angular momenta and gluon spin rather than by quark helicity [1,2]. Considerable effort has gone into understanding, predicting and measuring the corresponding transversity distribution of the nucleon constituents (see [3] and references therein). The leading twist transversity structure function, $h_1(x)$, is as fundamental to understanding the nature of the non-perturbative QCD regime of hadronic physics as is the longitudinal function $g_1(x)$, which in principle can be measured in hard scattering processes. Yet, the transversity distribution is suppressed at leading twist in deep inelastic lepton scattering since it is chiral odd. The same comparison applies to the various quark spin dependent helicity $\Delta q^a(x)$ and transversity $\delta q^a(x)$ (for flavor index a) distributions.

In their systematic study of the chiral odd distributions, Jaffe and Ji [4] related the first moment of the transversity distribution to the flavor contributions to the nucleon tensor charge:

$$\int_0^1 (\delta q^a(x) - \delta \bar{q}^a(x)) dx = \delta q^a. \quad (1)$$

Because there must be a helicity flip of the struck quark in order to probe the transverse spin polarization of the nucleon, the transversity distribution (and thus the tensor charge) decouples at leading twist in deep inelastic scattering. No such suppression appears in Drell-Yan scattering where Ralston and Soper [5] first encountered the transversity distribution entering the corresponding transverse spin (both the beam's and target's spin being transversely polarized to the incident beam direction) asymmetries. Consequently, the charge is difficult to measure and its non-conservation [6] makes it difficult to predict. While bounds placed on the leading twist quark distributions through positivity constraints suggest that they satisfy the inequality of Soffer [7];

$$|2\delta q^a(x)| \leq q^a(x) + \Delta q^a(x), \quad (2)$$

(where q^a denotes the unpolarized quark distribution), model calculations yield a range of theoretical predictions [3].

Like the isoscalar and isovector axial vector charges defined from the forward nucleon matrix element of the axial vector current, the nucleon tensor charge is defined from the corresponding forward matrix element of the tensor current,

$$\langle P, S_T | \bar{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | P, S_T \rangle = 2\delta q^a(\mu^2) (P^\mu S_T^\nu - P^\nu S_T^\mu), \quad (3)$$

where $S_T^Q \sim (|+> \pm |->)$ for the moving nucleon is the *transversity* [8]. Unlike the axial vector isovector charge, no sum rule has been written that enables a clear relation between the tensor charge and a low energy measurable quantity. So, aside from model calculations, there are no definitive theoretical predictions of

*Electronic mail: gamberg@dept.physics.upenn.edu†Electronic mail: goldste@tufts.edu

the tensor charge. Among the various approaches, from the QCD sum rule [9–11], which estimates the flavor contributions to the tensor charge by analyzing the bilocal tensor current, to lattice calculations [12] and light cone quark models [13], there appears to be a range of expectations and a disagreement concerning the sign of the d quark contribution.

Given the numerous experiments [14–17] and proposals [18,19] for extracting quark transversity distributions and the nucleon’s tensor charge, there is reason to expect that in the not too distant future one will be able to reliably compare the data to the theoretical estimates of these quantities.

Here we offer another estimate. Our motivation stems in part from the result that the tensor charge does not mix with gluons under QCD evolution and therefore behaves as a non-singlet matrix element. This, in conjunction with the fact that the tensor current is charge conjugation odd (it does not mix quark-antiquark excitations of the vacuum, since the latter is charge conjugation even, nor does it mix with gluonic operators under evolution, since gluonic operators are even), suggests that the tensor charge is more amenable to a valence quark model analysis. With this in mind we calculate the tensor charge by using axial vector dominance and an approximate phenomenological mass symmetry among the light axial vector mesons ($a_1(1260)$, $b_1(1235)$ and $h_1(1170)$). The b_1 and h_1 can couple to the quark tensor current in the nucleon at low energies, and via the symmetry, their coupling to the leptons is related to the a_1 production in τ decay. It is natural to represent this phenomenological mass degeneracy by $SU(6)_W \otimes O(3)$ spin-flavor symmetry. We utilize this symmetry in order to relate the parameters arising in the expressions for the tensor charge to measurable quantities. In the following we present our results for the isoscalar and isovector and, in turn, u and d quark contributions to the tensor charge of the nucleon and we compare our results with other model calculations. Finally we note a remarkable relation between the tensor charge and the nucleon axial vector coupling constant.

As mentioned above, we determine the tensor charge at a scale where the matrix element of the tensor current is dominated by the lowest lying axial vector mesons. [20] At this scale the matrix element of the tensor current, Eq. (3) is

$$\langle P, S_T | \bar{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | P, S_T \rangle = \sum_{\mathcal{M}} \frac{\langle 0 | \bar{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | \mathcal{M} \rangle \langle \mathcal{M}, P, S_T | P, S_T \rangle}{M_{\mathcal{M}}^2 - k^2}. \quad (4)$$

The summation is over those mesons with quantum numbers, $J^{PC} = 1^{+-}$ that couple to the nucleon via the tensor current; namely the charge conjugation odd axial vector mesons – the isoscalar $h_1(1170)$ and the isovector

$b_1(1235)$. To analyze this expression in the limit $k^2 \rightarrow 0$ we require the vertex functions for the nucleon coupling to the h_1 and b_1 meson and the corresponding matrix elements of the meson decay amplitudes which are related to the meson to vacuum matrix element via the quark tensor current. The former yield the nucleon coupling constants $g_{\mathcal{M}NN}$ defined from the matrix element

$$\langle MP | P \rangle = \frac{ig_{\mathcal{M}NN}}{2M_N} \bar{u}(P, S_T) \sigma^{\mu\nu} \gamma_5 u(P, S_T) \varepsilon_\mu P_\nu, \quad (5)$$

where P_μ is the nucleon momentum, and the latter yield the meson decay constant, $f_{\mathcal{M}}$

$$\langle 0 | \bar{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | \mathcal{M} \rangle = if_{\mathcal{M}}^a (\epsilon_\mu k_\nu - \epsilon_\nu k_\mu), \quad (6)$$

where the k_μ and ε_ν are the meson momentum and polarization respectively. For transverse polarized Dirac particles, $S^\mu = (0, S_T)$ we project out the tensor charge using the constraint on the vector meson, $k \cdot \varepsilon_{\mathcal{M}} = 0$

$$\delta q^a = \frac{f_{\mathcal{M}}^a g_{\mathcal{M}NN} (S_T \cdot k)^2}{2M_N M_{\mathcal{M}}^2}. \quad (7)$$

In order to evaluate the tensor charge at the scale dictated by the axial vector meson dominance model we must determine the isoscalar and isovector meson coupling constants. Taking a hint from the valence interpretation of the tensor charge, we exploit the phenomenological mass symmetry among the lowest lying axial vector mesons that couple to the tensor charge; we adopt the spin-flavor symmetry characterized by an $SU(6) \otimes O(3)$ [21,22] multiplet structure. Thus, the 1^{+-} h_1 and b_1 mesons fall into a $(35 \otimes L = 1)$ multiplet that contains $J^{PC} = 1^{+-}, 0^{++}, 1^{++}, 2^{++}$ states. This analysis enables us to relate the a_1 meson decay constant measured in $\tau^- \rightarrow a_1^- + \nu_\tau$ decay [23]

$$f_{a_1} = (0.19 \pm 0.03) \text{GeV}^2, \quad (8)$$

and the $a_1 NN$ coupling constant

$$g_{a_1 NN} = 7.49 \pm 1.0, \quad (9)$$

(as determined from a_1 axial vector dominance for longitudinal charge as derived by Birkel and Fritzsche [24] but using $g_A/g_V = 1.267$ [25]) to the meson decay constants, f_{b_1} , f_{h_1} and coupling constants, $g_{b_1 NN}$ and $g_{h_1 NN}$. We find

$$f_{b_1} = \frac{\sqrt{2}}{M_{b_1}} f_{a_1}, \quad g_{b_1 NN} = \frac{5}{3\sqrt{2}} g_{a_1 NN}, \quad (10)$$

where the $5/3$ appears from the $SU(6)$ factor $(1 + F/D)$ and the $\sqrt{2}$ arises from the $L = 1$ relation between the 1^{++} and 1^{+-} states. Our resulting value of $f_{b_1} \approx 0.21 \pm 0.03$ agrees well with a sum rule determination of 0.18 ± 0.03 [11,26]. The h_1 couplings are related to the b_1 couplings via $SU(3)$ and the $SU(6)$ F/D value,

$$f_{b_1} = \sqrt{3}f_{h_1}, \quad g_{b_1 NN} = \frac{5}{\sqrt{3}}g_{h_1 NN}. \quad (11)$$

These, in turn, enable us to determine the isovector and isoscalar parts of the tensor charge,

$$\delta q^v = \frac{f_{b_1}g_{b_1 NN}\langle k_\perp^2 \rangle}{\sqrt{2}M_N M_{b_1}^2}, \quad \delta q^s = \frac{f_{h_1}g_{h_1 NN}\langle k_\perp^2 \rangle}{\sqrt{2}M_N M_{h_1}^2}, \quad (12)$$

respectively (where, $\delta q^v = (\delta u - \delta d)$, and $\delta q^s = (\delta u + \delta d)$). Transverse momentum appears in these expressions because the tensor couplings involve helicity flips that carry kinematic factors of momentum transfer. The intrinsic k_\perp of the quarks in the nucleon is non-zero, as determined from Drell-Yan and heavy vector boson production processes. Using a Gaussian momentum distribution, and letting $\langle k_\perp^2 \rangle$ range from (0.58 to 1.0 GeV²) [27] results in the u and d quark transversity ranging from

$$\begin{aligned} \delta u(\mu^2) &= (0.43 \text{ to } 0.74) \pm 0.20, \\ \delta d(\mu^2) &= -(0.26 \text{ to } 0.47) \pm 0.20. \end{aligned} \quad (13)$$

These values for the u -quark tensor charge lie slightly lower than most other estimates while the d -quark charge is negative and of a comparable magnitude. These results, with their uncertainty, are consistent with the Soffer inequality [7] applied to the charges, $q + \Delta q \geq 2|\delta q|$, although near the equality. Note that many predictions have the ratio $\delta d/\delta u$ near $-1/4$ or $(1 - \sqrt{3})/(1 + \sqrt{3})$, the value resulting from an $SU(3)$ degeneracy between the π^0 and the $\eta(8)$ octet elements in their coupling to the u -quark and the d -quark, i.e. the isoscalar coupling to u and d -quarks is $1/\sqrt{3}$ times the isovector. In our calculation the isoscalar and isovector axial vector couplings to the nucleon also enter as factors in the expressions for the charges, with the D/F ratio being $3/2$ in exact $SU(6)$. Loosening the $SU(6)$ constraint and incorporating mixing of the $h_1(1170)$ with the $h_1(1380)$ will alter the u to d ratio.

In relating the $b_1(1235)$ and $h_1(1170)$ couplings in Eq. (11) we assumed that the latter isoscalar was a pure octet element, $h_1(8)$. Experimentally, the higher mass $h_1(1380)$ was seen in the $K + \bar{K} + \pi's$ decay channel [25,28] while the $h_1(1170)$ was detected in the multi-pion channel [25,29]. This decay pattern indicates that the higher mass state is strangeonium and decouples from the lighter quarks – the well known mixing pattern of the vector meson nonet elements ω and ϕ . If the h_1 states are mixed states of the $SU(3)$ octet $h_1(8)$ and singlet $h_1(1)$ analogously, then

$$\begin{aligned} h_1(1170) &= \sqrt{\frac{2}{3}}h_1(8) - \frac{1}{\sqrt{3}}h_1(1), \\ h_1(1380) &= \frac{1}{\sqrt{3}}h_1(8) + \sqrt{\frac{2}{3}}h_1(1), \end{aligned} \quad (14)$$

from which it follows that

$$f_{h_1(1170)} = f_{b_1}, \quad \text{and} \quad g_{h_1(1170)NN} = \frac{3}{5}g_{b_1 NN}, \quad (15)$$

with the $h_1(1380)$ not coupling to the nucleon (for $g_{h_1(1)NN} = \sqrt{2}g_{h_1(8)NN}$). These symmetry relations alter the results in Eq. (13) to

$$\begin{aligned} \delta u(\mu^2) &= (0.58 \text{ to } 1.01) \pm 0.20, \\ \delta d(\mu^2) &= -(0.11 \text{ to } 0.20) \pm 0.20. \end{aligned} \quad (16)$$

The values in Eq. (16) are closer to several other model calculations including: tensor charges on the lattice [12]; QCD sum rules [9], the bag model [30,9] and a numerically similar Melosh Transform approach [13]; the chiral quark model with Goldstone boson effects [31]; and quark soliton models [32–34].

The calculation has been carried out at the scale $\mu \approx 1$ GeV, which is set by the nucleon mass as well as being the mean mass of the axial vector meson multiplet. The appropriate evolution to higher scales (wherein the Drell-Yan processes are studied) is determined by the anomalous dimensions of the tensor charge [6] which is straightforward but slowly varying.

It is interesting to observe that the symmetry relations that connect the b_1 couplings to the a_1 couplings in Eq. (10) can be used to relate directly the isovector tensor charge to the axial vector coupling g_A . This is accomplished through the a_1 dominance expression for the isovector longitudinal charges derived by Birkel and Fritzsche [24],

$$\Delta u - \Delta d = \frac{g_A}{g_V} = \frac{\sqrt{2}f_{a_1}g_{a_1 NN}}{M_{a_1}^2}. \quad (17)$$

Hence for δq^v we have

$$\delta u - \delta d = \frac{5}{6} \frac{g_A}{g_V} \frac{M_{a_1}^2}{M_{b_1}^2} \frac{\langle k_\perp^2 \rangle}{M_N M_{b_1}}, \quad (18)$$

a remarkable relation that appears more fundamental than its derivation. It is important to realize that this relation can hold at the scale wherein the couplings were specified, the meson masses, but will be altered at higher scales (logarithmically) by the different evolution equations for the Δq and δq charges. To write an analogous expression for the isoscalar charges ($\Delta u + \Delta d$) would involve the singlet mixing terms and gluon contributions, as Ref. [24] considers. However, given that the tensor charge does not involve gluon contributions (and anomalies), it may be expected that the relation between the h_1 and b_1 couplings in the same $SU(3)$ multiplet will lead to a more direct result

$$\delta u + \delta d = \frac{3}{5} \frac{M_{b_1}^2}{M_{h_1}^2} \delta q^v, \quad (19)$$

for the ideally mixed singlet-octet $h_1(1170)$. These are also remarkable relations that are quite distinct from other predictions.

In conclusion, our axial vector dominance model with $SU(6)_W \otimes O(3)$ coupling relations provide simple formulae for the tensor charges. This simplicity belies the considerable subtlety of the (non-perturbative) hadronic physics that is summarized in those formulae. We obtain the same order of magnitude as most other calculation schemes. These results support the view that the underlying hadronic physics, while quite difficult to formulate from first principles, is essentially a 1^{+-} meson exchange process. Forthcoming experiments will begin to test this notion.

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